EQUALIZATION FOR TRANSMISSION LINE CHANNELS:

A DISCUSSION OF THREE IIR ADAPTIVE FILTERING ALGORITHMS

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ABSTRACT

The performances of two low-complexity IIR adaptive filtering algorithms applied to the equalization of a typical transmission line channel are compared. The algorithms considered are the Simple Hyperstable Adaptive Recursive Filter (SHARF) and the stochastic gradient equation error algorithm. Gradient descent techniques are shown to be ill suited for transmission line channels.

INTRODUCTION

The transmission of digital data through a transmission line channel, i.e., twisted wire pair or coaxial cable, results in intersymbol interference at the receiving end, due mainly to channel amplitude distortion. In addition, the channel may vary slowly with time due to temperature changes or as in the case of switched telephone lines due to the fact that the channel is connection dependent. Analog adaptive equalizers have been used in this context with some degree of success [1]. They are simple to implement with an automatic gain control circuit and a zero shifting network and provide continuous adaptation. With the advent of inexpensive digital signal processors, however, consideration should be given to adaptive digital equalizers. Adaptive filtering is a mature subject whose results can be used in the design of equalizers with potential gains in both flexibility and performance. Since the parameters of the channels under consideration are slowly varying compared to the transmisson rates we will consider only transient adaptive digital equalizers. The adaptation of the equalizer will take place during a start up time using a known training signal, commonly a pseudo-random binary sequence, available at both ends of the channel.

LINEAR EQUALIZATION

A block diagram of the equivalent discrete-time linear filter model for the channel and equalizer is shown in Fig. 1. Ideally, the equalizer with

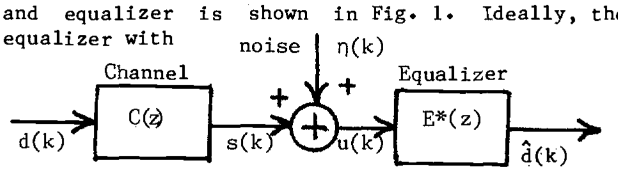

Fig. 1 Block diagram of channel model and equalizer.

transfer function $E^*(z)$, is simply the inverse filter of the linear filter model $C(z)$, i.e.,

$$E^*(z) = \frac{1}{C(z)} \tag{1}$$

A general form for $C(z)$ is the ARMA structure

$$C(z) = \frac{A(z)}{B(z)} \tag{2}$$

leading to an equalizer of the form

$$E^*(z) = \frac{B(z)}{A(z)} \tag{3}$$

Some restrictions must be imposed on the channel model $C(z)$. It must be proper rational, so that $E^*(z)$ is causal, stable and minimum phase.

Although some work on adaptive implementations of equalizers having an ARMA structure has been reported [2], most of the work has been concentrated on fast transversal equalizers [3].

TRANSMISSION LINE CHARACTERIZATION

Typically, the baseband transmission medium is either balanced cable pair, like a twisted wire pair, or unbalanced cable pair such as coaxial cable. As the length between transmitter and receiver increases, pulse dispersion into adjacent time slots occurs due to losses in the line. Diverse and unique cable transmission characteristics result from different cable construction. A common frequency response model for coaxial cables is given by [4]

$$L(f) = \exp\{-\alpha_0\sqrt{f/f_N} - j\alpha_0\sqrt{f/f_N}\} \tag{4}$$

where f is the frequency in Hz, and α_o is the line attenuation, in nepers, at the Nyquist frequency f_N. The Nyquist frequency f_N is defined as half the signalling frequency f_s.

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Since for most baseband systems pulse dispersion is caused almost entirely by channel amplitude distortion [1], an equalizer which approximates the inverse of these channel characteristics will suffice. One commonly used approach is to approximate the line's characteristics by a first order linear system C(s), over the frequency range for which the transmitted signal has significant energy, usually up to 1.7 of the Nyquist frequency [1]. The simplified model of the line, normalized to the Nyquist frequency, is given by

$$C(s) = \frac{1}{1 + (s/f_n)z_o} \tag{5}$$

The ideal linear equalizer would be of the form

$$E^*(s) = 1 + (s/f_N)z_o \tag{6}$$

To examine the validity of such an approximation, two examples are presented. The criterion used for pole location is that both L(f) and C(f) have the same attenuation at the Nyquist frequency. For the first example [1], the pole of the first order approximation is set at 26.5 kHz, yielding an attenuation of about 53 dB at the assumed Nyquist frequency of 570 kHz. Simulation results for this example are shown in Fig. 2, where the response of the equalizer given by (8) has been offset to intersect the other curves at f_N. The second example [4], assumes f_N =52.5 MHz and a corresponding attenuation of 84 dB, which gives a pole at about 417 kHz for the linear approximation. The simulation results for this example are given in Fig. 3. It can be seen from the given examples that the equalized signal is relatively constant and smooth over the desired frequency range, and the linear approximation can be deemed acceptable.

Since our objective is to investigate the feasibility of using a digital adaptive equalizer, a discrete-time equivalent for the line's model is necessary. Among the various methods available [5], the bilinear transformaton was selected. The generic discrete-time equivalent of (5) is obtained by transforming the normalized analog frequency, to give

$$C_d(z) = \frac{1}{1 + \left(\frac{1-z^{-1}}{1+z^{-1}}\right)z_o} \tag{7}$$

which leads to an ideal equalizer of the form

$$E_d^*(z) = \frac{(1+z_o) + (1-z_o)z^{-1}}{1 + z^{-1}} \tag{8}$$

It should be noted that the equalizer has an ARMA structure. This clearly suggests that the transversal equalizer is not the best suited structure for a transmission line channel.

ADAPTIVE FILTERING ALGORITHMS

As long as the approximate model of (5) adequately describes the line characteristics, the equalization problem is reduced to the identification of (8) from input/ouput line data. Since no fast variations are experienced by the line parameters, continuous adaptation and fast converging algorithms are not needed. Our interest will focus on low complexity algorithms. Three algorithms will be considered: i) the recursive gradient descent algorithm based on output error minimization [6]-[9], ii) the equation-error minimization algorithm [10]-[13], and iii) a simplified algorithm based on hyperstability considerations [2], [14], [15].

A block diagram of the adaptive equalizer is shown in Fig. 4. During the training period the transmitted signal will be available at the receiving end, and will be used as the desired signal by the adaptive equalizer. The equlization task become simply an identification problem.

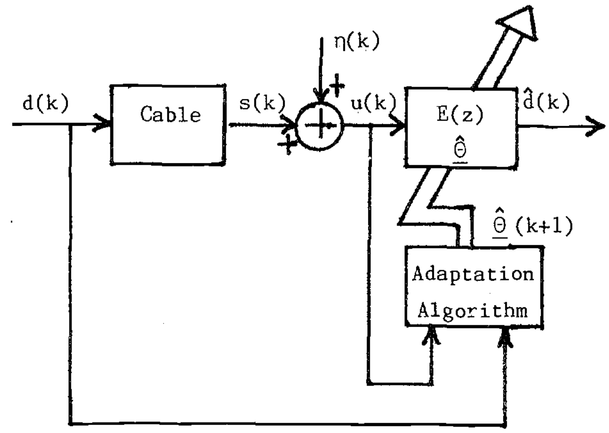

Fig. 4 Block diagram of adaptive equalizer.

The ARMA identifier will be of the form

$$\hat{d}(k) = \sum_{j=0}^{M} \hat{b}_j(k)u(k-j) + \sum_{i=1}^{N} \hat{a}_i(k)\hat{d}(k-i) \tag{9}$$

or in matrix notation

$$\hat{d}(k) = \underline{\hat{\Theta}}^T(k)\underline{X}(k) = \underline{X}^T(k)\underline{\hat{\Theta}}(k) \tag{10}$$

where

$$\underline{\hat{\Theta}}(k) = [\hat{b}_o(k)\ldots\hat{b}_M(k) \mid \hat{a}_1(k)\ldots\hat{a}_N(k)]^T \tag{11.a}$$

and

$$\underline{X}(k) = [u(k)\ldots u(k-M) \mid d(k-1)\ldots d(k-N)]^T \tag{11.b}$$

are the parameter and information vector, respectively. Using the delay operator notation, (10) can be rewritten as

$$\hat{d}(k) = \hat{B}_o^M(k)u(k) + \hat{A}_1^N(k)\hat{d}(k) \quad (12)$$

or

$$\frac{\hat{d}(k)}{u(k)} = \frac{\hat{B}_o^M(k)}{1-\hat{A}_1^N(k)} \quad (13)$$

where

$$\hat{B}_o^M(k) = \hat{b}_o(k) + \hat{b}_1(k)q^{-1} + \ldots + \hat{b}_M(k)q^{-M} \quad (14)$$

and

$$\hat{A}_1^N(k) = \hat{a}_1(k)q^{-1} + \hat{a}_2(k)q^{-2} + \ldots + \hat{a}_N(k)q^{-N} \quad (15)$$

are the delay operator polynomials and $q^{-\ell}$ represents a delay of ℓ samples. The output error is defined as

$$e(k) = d(k) - \hat{d}(k) = \frac{[1 - \hat{A}_1^N(k)]d(k) - \hat{B}_o^M(k)u(k)}{1- \hat{A}_1^N(k)} \quad (16)$$

The descent methods of [6]-[9] seek to minimize the square of (15), i.e., find a local estimate for the mean squared error (MSE). It can be seen from (16) that such minimization will be nonlinear in the autoregressive coefficients of (9). The MSE surface corresponding to (16) will be, in general, nonconvex or multimodal [9], and the gradient search method may lead to local minimization. In [6] and [7] auxiliary recursive processes are obtained to compute the gradients of the square of (16) with respect to the MA and AR coefficients of the identifier. This technique, however, leads to an increase in the computational load of the algorithm. Furthermore, instability may occur due to the recursive nature of the auxiliary processes, thus requiring on-line monitoring and correction techniques [15]. These are undesirable characteristics that prevent the use of such algorithms when low-complexity is the main consideration. Similar algorithms have been proposed in the communications literature under the designation of MSE equalizers [16]. These algorithms reduce to the type discussed above when an infinite number of taps is assumed for an MA equalizer.

The equation error is obtained as a generalized error [10], by passing (16) through a filter, giving

$$\xi(k) = [1 - \hat{A}_1^N(k)]d(k) - \hat{B}_o^M(k)u(k) = d(k) - [\hat{A}_1^N(k)d(k) + \hat{B}_o^M(k)u(k)] \quad (17)$$

It can be viewed as the output error between d(k) and a two input, one output transversal filter given by the bracketed term in (17), [13], [15]. Since (17) is linear in both the AR and MA coefficients of (9), its minimization is a much more amenable task. In addition, if $\xi(k)=0$, e(k) will also vanish. By using steepest descent to search for the unique minimum of the squared error surface corresponding to (17), the following extrapolation of the LMS algorithm [17] is obtained

$$\hat{b}_j(k+1) = \hat{b}_j(k) + 2\mu\xi(k)u(k-j) \quad j=0,1,\ldots,M \quad (18.a)$$

$$\hat{a}_i(k+1) = \hat{a}_i(k) + 2\mu\xi(k)d(k-i) \quad i=1,2,\ldots,N \quad (18.b)$$

where μ is the step size, $\mu \in R$, $\mu > 0$. If instead of u(k), perfect measurements of s(k) were available, then the estimates of (18) would converge in the mean to the true parameters of the ideal equalizer, [12], [13], provided that

$$0 < \mu < \frac{1}{\underline{X}^T(k)\underline{X}(k)} \quad \forall k \in N \cup \{0\} \quad (19)$$

However, only noisy measurements u(k) are available, and thus bias in the estimates will appear, precluding in turn a global minimization of (16). The algorithm of (18) will be referred to as the extended LMS (ELMS) algorithm. Similar algorithms have been developed in the cmmunications literature and are termed decision-feedback algorithms, [16], [18]. These algorithms reduce to the ELMS algorithm when the feedback symbols are correct.

The hyperstable recursive filters are perhaps the most elegant solution to the adaptive infinite impulse response (IIR) filtering problem currently available. The hyperstable solution to recursive adaptive filtering formulated by Johnson [14], can be greatly simplified when the estimates vary slowly from iteration to iteration. Under this assumption, an adaptation algorithm known as SHARF [2], has been proposed. The parameter equations for this algorithm are given by

$$v(k) = e(k) + \sum_{\ell=1}^{p} c_\ell e(k-\ell) \quad (20.a)$$

$$\hat{b}_j(k+1) = \hat{b}_j(k) + \zeta_j v(k)u(k-j) \quad j=0,1,\ldots,M \quad (20.b)$$

$$\hat{a}_i(k+1) = \hat{a}_i(k) + \mu_i v(k)\hat{d}(k-1) \quad i=1,2,\ldots,N \quad (20.c)$$

where ζ_j and μ_i are positive step sizes and e(k) is defined as (16). A sufficient, and thus possible over restrictive, condition for convergence is that the smoothing coefficients of (20.a) be chosen to meet the following strictly positive reality (SPR) condition

$$Re\left\{ \frac{1+\sum_{\ell=1}^{p} c_\ell z^{-\ell}}{1-\sum_{i=1}^{N} a_i z^{-i}} \right\} \Bigg|_{\substack{z=e^{j\theta} \\ \theta\varepsilon[0,2\pi]}} > 0 \qquad (21)$$

where the a_i 's are the true autoregressive coefficients of the ideal linear equalizer. Let us examine the validity of this condition for the equalizer of (8). The bracketed expression in (21) becomes

$$G(z) = \frac{1 + \sum_{\ell=1}^{p} c_\ell z^{-\ell}}{1- (-1)\ z^{-1}} \qquad (22)$$

Hence, by selecting p=1, c_1=1, G(z) is made to be trivially SPR. Furthermore, if all the smoothing coefficients are selected to be zero, then (21) becomes

$$Re\left\{ \frac{1}{1-(-1)z^{-1}} \right\} \Bigg|_{z=e^{j\theta}} = Re\left\{ \frac{e^{j\theta}}{1+e^{j\theta}} \right\} = \tfrac{1}{2} > 0 \qquad (23)$$

this shows that the discrete-time equivalent of (6) is inherently SPR. The same result would have been obtained had we used an impulse invariance transformation [5] instead of the bilinear transformation.

SIMULATION RESULTS

The performance of the ELMS and SHARF algorithms for a typical baseband transmission system was investigated. For comparison purposes a 20 tap transversal equalizer using the LMS algorithm was also implemented. For the SHARF algorithm, no smoothing was provided, so its performance is very similar to the ELMS algorithm. The step size was chosen to be μ=0.1 for all simulation runs, and the training sequence was a maximum length null sequence negative of order 11, [19].

CONCLUSIONS

The feasibility of using digital adaptive filtering algorithms in the equalization of transmission line channels has been discussed. By explicitly viewing the equalization problem as a system identification problem, some connections between research efforts arising from seemingly different approaches were suggested. Simulation results comparing the performance of different adaptation algorithms used in the equalization of a typical transmission line channel were presented.

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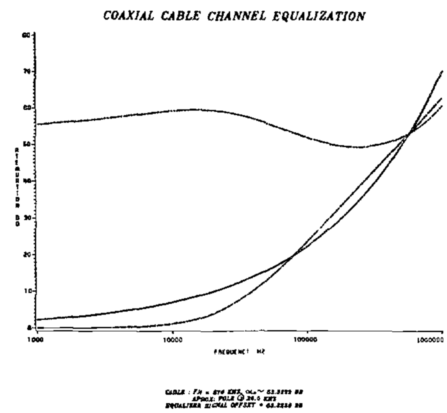

Fig. 2 Simulated Channel Attenuation: _____ L(f), ---C(f).

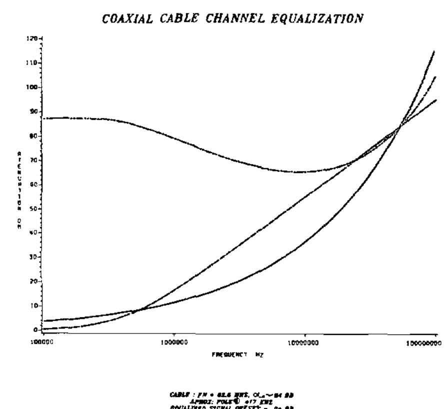

Fig. 3 Simulated Channel Attenuation: _____L(f), ---C(f).

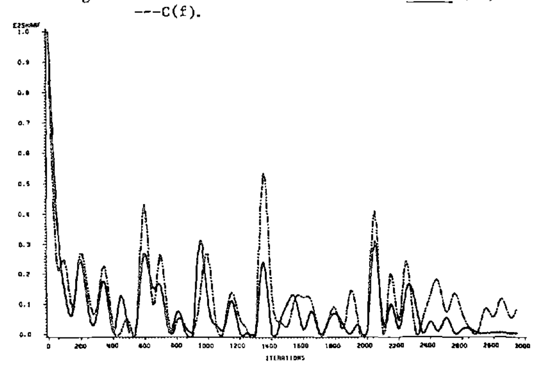

Fig. 5 Learning curves for a signal-to-noise ratio of 30dB: -·-·-·-SHARF, _____ELMS.