



# Linear quadratic dynamic programming for water reservoir management

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*Dynamic programming (DP) is applied in order to determine the optimal management policy for a water reservoir by modeling the physical problem via a linear quadratic (LQ) structure. A simplified solution to the LQ tracking problem is provided under mild assumptions. The model presents an aggregated multicriteria decision making problem where flood control, hydroelectric power, and water demand have to be satisfied: Simultaneously the energy production is to be maximized, the mismatch of water demand minimized, and the water release should not cause flooding. The system constraints are basically the conservation of mass within the reservoir system, and the minimum and the maximum allowable limits for the water release and the reservoir level. The stochastic variables consist of the water inflow from the reservoir drainage basin, precipitation, and evaporation. The Tenkiller Ferry dam on the Illinois River basin in Oklahoma is analyzed as a case study. © 1997 by Elsevier Science Inc.*

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## 1. Introduction and scope

The purpose of this study is to solve a stochastic reservoir control problem by means of a linear system model and quadratic cost (LQ) framework. There has been extensive research on reservoir operation using a dynamic programming (DP) methodology. Yakowitz<sup>1</sup> gave a comprehensive review of DP models for water resource problems and examined computational techniques that have been used to obtain solutions to these problems. DP applications in reservoir management was reviewed by Yeh<sup>2</sup>. A taxonomic scheme for grouping the DP applications in water resource systems can also be found in Esogbue.<sup>3</sup>

The LQ framework has been widely used in control systems, mainly because of the structural advantages of the problem definition. In LQ problems the computations are relatively simple after the theoretical development and the problems due to dimensionality are overcome. Some of the literature related to the work presented in this paper is as follows: Pindyck<sup>4</sup> has provided the solution for the LQ tracking problem, which was used for policy planning in economical systems. Chan and Maille<sup>5</sup> extended Pindyck's solution for problems where magni-

tude constraints on the controls exist. Jacobson,<sup>6</sup> and Jacobson et al.<sup>7</sup> worked on the extensions of LQ theory. Wasimi and Kitanidis<sup>8</sup> are the first to apply discrete-time LQ Gaussian (LQG) stochastic control techniques to a reservoir system under flood conditions to optimize its operation such that the expected value of flood damage is minimized during a relatively short operating horizon and yet is consistent with the long-term operating strategy. Loaiciga and Marino<sup>9</sup> used LQG with maximum likelihood estimators for the system parameters. Georgakakos and Marks<sup>10</sup> used a quadratic programming approach, which they named extended LQG (ELQG), mainly because it can be used to solve LQG problems as well as systems with nonlinear dynamics, control and reliability state constraints, and nonquadratic performance indices. Georgakakos<sup>11</sup> illustrated further extensions to deal with the non-Gaussian features that frequently characterize reservoir inputs. McLaughlin and Velasco<sup>12</sup> applied LQ control to a reservoir system in the Caroni River basin of Venezuela.

In this paper an alternative solution for a specific case of LQ tracking problem is presented. The solution makes use of the well-known linear optimal feedback control law,<sup>13-15</sup> after a proper transformation of the LQ "tracking problem" into an LQ "regulation problem" structure. The methodology is illustrated with a computational experiment for the Tenkiller water dam on the Illinois River basin in Oklahoma. Our aim is to find the optimal management policy, in such a way that the hydroelectric

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power, flood control, and water demands are satisfied, and also to give a comparison with actual historical operation data.

In the next sections we review the general LQ model and the description of the stationary solution, and we present the simplified stationary LQ tracking problem solution. Then the application case is provided. Finally the results are summarized and conclusions are drawn.

## 2. Methodology

### 2.1. Basic problem

In stochastic dynamic programming problems at each stage we select a decision such that it minimizes the expected value of the sum of the current stage cost and the optimal total cost that can be expected from future stages. In this study we work with discrete-time systems in which the state process is described by the equation

$$x_{k+1} = f_k[x_k, u_k, w_k], \quad k = 0, \dots, K-1 \quad (1)$$

where the state  $x_k$  is an element of state space  $S_k$ , the control  $u_k$  is an element of a decision (or control) space  $C_k$ , and the random disturbance  $w_k$  is an element of the space  $D_k$ . The control  $u_k$  is constrained, and it takes values from a known (and nonempty) subset  $U_k(x_k)$  of  $C_k$ , which is dependent on the current state  $x_k$ . The random disturbance  $w_k$  depends explicitly on  $x_k$  and  $u_k$  but not the previous  $w_{k-1}, \dots, w_0$  values, and it is characterized by a probability measure  $P_k(\cdot|x_k, u_k)$ . Initially we consider the case of a finite planning horizon ( $K < \infty$ ). For this case the class of *admissible* control laws (or policies) consists of sequences of functions  $\pi = \{\mu_0, \mu_1, \dots, \mu_{K-1}\}$ , where  $\mu_k$  is a decision function such that it maps states  $x_k$  into controls  $u_k = \mu_k(x_k) \in U_k(x_k)$ .

For an initial state  $x_0$  the problem is to find an admissible control law  $\pi = \{\mu_0, \mu_1, \dots, \mu_{K-1}\}$ ,  $\mu_k: S_k \rightarrow C_k$  such that it minimizes the cost functional

$$J_\pi(x_0) = E_{w_k} \left\{ g_K(x_K) + \sum_{k=0}^{K-1} g_k[x_k, \mu_k(x_k), w_k] \mid x_0 \right\}, \quad \forall x_0 \quad (2)$$

subject to the state equation (1). The one-stage cost functions  $g_k$  are assumed to be given. An optimal control law  $\pi^*$  minimizes the corresponding total cost

$$J_{\pi^*}(x_0) = \min_{\pi \in \Pi} J_\pi(x_0), \quad \forall x_0 \quad (3)$$

where  $\Pi$  denotes the set of all admissible laws.

### 2.2. The general unconstrained LQ model

Here we will briefly state the general formulation and results for the LQ problems (for more details the reader is

referred to Bertsekas<sup>15</sup>. An LQ problem consists of a linear system equation model and a quadratic cost functional. The state equation is given as

$$x_{k+1} = A_k x_k + B_k u_k + w_k \quad (4)$$

where  $x_k$  and  $u_k$  are vectors of dimension  $n$  and  $m$ , respectively, and  $A_k$  and  $B_k$  are given matrices with appropriate dimensions. The objective is to find a control law that minimizes the quadratic cost function

$$E_{w_k} \left\{ x'_K Q_K x_K + \sum_{k=0}^{K-1} (x'_k Q_k x_k + u'_k R_k u_k) \mid x_0 \right\}, \quad \forall x_0 \quad (5)$$

where  $Q_k$  and  $R_k$  are given matrices with appropriate dimensions. The selection of  $Q_k$ 's and  $R_k$ 's takes into consideration the design objectives. If there are no restrictions for the values that decision  $u_k$  can take, then it is called an unconstrained model. For the validity of the model it is assumed that  $Q_k$ 's are symmetric, positive semi-definite matrices and  $R_k$ 's are symmetric and positive definite (see e.g., Athans<sup>14</sup>). Moreover the disturbances  $w_k$  are independent random vectors with zero mean and finite second moment, and their given probability distribution does not depend on either the states or the controls.

If we want to describe a problem where the target has a nonzero value, we obtain the so-called "tracking" or "regulation" problem, where we try to follow the schedule:  $\{\bar{x}_k\}$ . Equation (5) then becomes

$$E_{w_k} \left\{ (x_K - \bar{x}_K)' Q_K (x_K - \bar{x}_K) + \sum_{k=0}^{K-1} ((x_k - \bar{x}_k)' Q_k (x_k - \bar{x}_k) + u'_k R_k u_k) \mid x_0 \right\}, \quad \forall x_0 \quad (6)$$

The reasons behind using a quadratic objective function are 3 fold: (1) Physically we want to keep the state of the system close to a given target level, so we give a high penalty for large deviations of the state from the target, but a relatively small penalty for small deviations; (2) mathematically, to justify the linear system approximation of a nonlinear system, second- and higher-order terms in the Taylor series expansion need to be minimized, which can be represented as the minimization of a quadratic function<sup>16</sup>; and (3) quadratic functions enable us to obtain analytical solutions for the optimal control using objective problem (6). Applying the DP algorithm leads to the following linear optimal feedback control law (see e.g., Bertsekas<sup>15</sup> and Kirk<sup>13</sup>)

$$\mu_k^*(x_k) = L_k(x_k - \bar{x}_k), \quad \forall k \quad (7)$$

where  $L_k$ 's are "gain matrices" with the form

$$L_k = -(B'_k M_{k+1} B_k + R_k)^{-1} B'_k M_{k+1} A_k \quad (8)$$

and  $M_k$ 's are symmetric and positive semi-definite matrices recursively computable as

$$M_N := Q_N \quad (9)$$

$$M_k := A'_k \left[ M_{k+1} - M_{k+1} \times B_k (B'_k M_{k+1} B_k + R_k)^{-1} B'_k M_{k+1} \right] A_k + Q_k \quad (10)$$

Equation (10) is called the discrete-time Riccati equation.<sup>15</sup> The optimal cost has the form:

$$J_0(x_0) = (x_0 - \underline{x}_0)' M_0 (x_0 - \underline{x}_0) + \sum_{k=0}^{N-1} E\{w'_k M_{k+1} w_k\} \quad (11)$$

where  $J_0(x_0)$  depends on  $x_0$ , the statistics of  $\{w_k\}$ , and design parameters  $R_k$  and  $Q_k$ .

### 2.3. Stationary solution to the general LQ problem

If the  $A_k$ ,  $B_k$ ,  $Q_k$ , and  $R_k$  matrices do not depend on the time index  $k$ , then the solution for equation (10) converges to a steady-state solution  $M$ , which satisfies the equation<sup>15</sup>

$$M := A' [M - MB(B'MB + R)^{-1} B'M] A + Q \quad (12)$$

the so-called algebraic Riccati equation. The consequence of this property is that if the number of stages is large, i.e.,  $K \rightarrow \infty$ , then we can approximate the control law by a linear stationary control law where we apply the same policy for each time step:  $\mu^*, \mu^*, \dots, \mu^*$ , where

$$\mu^*(x) = L(x - \underline{x}) \quad (13)$$

$$L = -(B'MB + R)^{-1} B'MA \quad (14)$$

and  $M$  is the steady-state solution of the algebraic Riccati equation (12).

### 2.4. LQ with policy tracking

Let us consider a more generalized case of equation (6), where it is also desired to keep the control  $u_k$  at a target level  $\underline{u}_k$ . This situation might arise for example in water reservoir systems, where one would want to meet contracted energy demand and water demand. Equation (6)

can then be reformulated as follows:

$$E_{w_k} \left\{ (x_K - \underline{x}_K)' Q_K (x_K - \underline{x}_K) + \sum_{k=0}^{K-1} (x_k - \underline{x}_k)' Q_k (x_k - \underline{x}_k) + (u_k - \underline{u}_k)' R_k (u_k - \underline{u}_k) \mid x_0 \right\}, \forall x_0 \quad (15)$$

As mentioned before this problem has been analyzed by Pindyck<sup>4</sup> and then by Chan and Maille,<sup>5</sup> and their solutions make use of the Belman-Hamiltonian equation<sup>13</sup> to get an analytical linear feedback solution. Here an alternative solution for a specific case of the tracking problem is presented that makes use of the existing "regulation problem" results presented in Section 2.2. The results of this section are useful in some physical systems, as will be demonstrated later by means of a case study for a reservoir management problem. Furthermore our formulation allows to model  $\{w_k\}$  with a nonzero mean, which is usually the case for physical systems. We need the following assumptions.

**Assumption A.**  $B_{k+1} \underline{u}_{k+1} = B_k \underline{u}_k$ .

The above assumption can be achieved when the target  $\underline{u}_k$  and "control matrix"  $B_k$  do not vary with time. As will be shown later, for the reservoir control problem case study, if a daily control law is to be developed for a given month and/or season, time invariancy of the control target  $\underline{u}_k$  becomes reasonable. Moreover for our case study  $B_k$  is the identity matrix.

**Assumption B.**  $A_k$  is equal to an identity matrix with appropriate dimensions.

Clearly Assumption B might be restrictive in general, however, for the reservoir control case study that we analyze subsequently, the problem dynamics allows us to validate this assumption. Furthermore this assumption holds in most inventory control problems.

**Lemma 1.** If Assumptions A and B hold, then the objective function given by equation (15) subject to the state equation (4) can be equivalently rewritten in the form of equation (6) as

$$E_{w_k} \left\{ (\bar{x}_K - \bar{x}_K)' Q_K (\bar{x}_K - \bar{x}_K) + \sum_{k=0}^{K-1} ((\bar{x}_k - \bar{x}_k)' Q_k (\bar{x}_k - \bar{x}_k) + \bar{u}'_k R_k \bar{u}_k) \mid \bar{x}_0 \right\} \quad (16)$$

subject to the transformed state equation

$$\bar{x}_{k+1} = \bar{x}_k + B_k \bar{u}_k + \bar{w}_k \quad (17)$$

where

$$\bar{x}_k = x_k - k(\bar{w} + B_k \underline{u}_k) \quad (18)$$

$$\bar{x}_k = \underline{x}_k - k(\bar{w} + B_k \underline{u}_k) \quad (19)$$

$$\bar{u}_k = u_k - \underline{u}_k \quad (20)$$

$$\bar{w}_k = w_k - \bar{w} \quad (21)$$

$\bar{x}_k$  is the transformed state trajectory, and  $\bar{w}$  is the mean of the system noise.

*Proof:*

By substituting equations (18), (20), and (21) into equation (17) we get

$$\begin{aligned} x_{k+1} - (k+1)(\bar{w} + B_{k+1} \underline{u}_{k+1}) \\ = x_k - k(\bar{w} + B_k \underline{u}_k) + B_k(u_k - \underline{u}_k) + (w_k - \bar{w}) \end{aligned} \quad (22)$$

Since  $B_{k+1} \underline{u}_{k+1} = B_k \underline{u}_k$  we obtain

$$x_{k+1} = x_k + B_k u_k + w_k \quad (23)$$

which completes the proof.

**Corollary.** The system described by equations (16)–(21) has the following optimal linear feedback policy:

$$\begin{aligned} \bar{u}_k = \mu_k^*(\bar{x}_k) = L_k(\bar{x}_k - \bar{x}_k), \forall k \\ = L_k(x_k - \underline{x}_k) \end{aligned} \quad (24)$$

where  $L_k$  is the gain matrix from the corresponding discrete-time Riccati equation (10). Furthermore, when  $K \rightarrow \infty$ , steady-state solutions (i.e., equations [12]–[14]) are valid.

*Proof:*

The formulations given in equations (16) and (17) have the same structure as the LQ formulation given in equation (6), therefore all the results presented before hold for the transformed problem.

### 3. Application: Reservoir control

#### 3.1. Physical system

In many natural resource management processes conservation of mass principle plays an important role. This principle basically states that the change in the storage is equal to the difference of input and output. If we assume a linear storage relation, then the conversation of mass principle can be stated more formally as

$$x_{k+1} = x_k + i_k - o_k \quad (25)$$

where  $x_k$  is the state of the system, and  $i_k$  and  $o_k$  are the input and output to the system at time  $k$ , respectively. For

a reservoir system we have the following generalization of equation (25):

$$\begin{aligned} v_{k+1} = \max\{\min\{v_{max}, v_k \\ + (f_k + y_k - e_k) - r_k\}, v_{min}\}, \\ r_k \in (r_{min}, r_{max}) \end{aligned} \quad (26)$$

where the  $k$  subscript stands for time, usually in the order of days, weeks, or months;  $v_k$  denotes the storage level (the state of the system);  $v_{max}$  denotes the maximum storage or capacity of the reservoir (maximum allowed value of the state);  $f_k$  denotes inflow from the tributaries (a stochastic variable);  $y_k$  denotes precipitation on the dam (a stochastic variable);  $e_k$  denotes evaporation (a stochastic variable);  $r_k$  denotes the amount of water released (the decision variable);  $r_{max}$  denotes the maximum amount of water allowed for release (e.g., for flood control); and  $r_{min}$  denotes the minimum amount of water released (for recreational and agricultural needs, etc.).

#### 3.2. LQ model

The objective, as mentioned before, is to find the optimal management policy. Let us assume that to satisfy the demands for recreational purposes and to regulate the pumping rates in an irrigation district, the reservoir should have a certain storage, say  $v_T^k$ , where  $T$  stands for target, and similarly we try to keep the water release at  $r_T^k$  for agricultural needs downstream, fulfilling the hydroelectric power demand and/or minimizing flood damages. One can take the deviations of the actual values at each time  $k$  as a cost and try to minimize it. Let  $\alpha > 0$  and  $\beta > 0$  be weights for the costs resulting from storage and release demands, respectively. Assume that the scaling factor to bring the storage and release values to the same order of magnitude is already included in  $\alpha$  and  $\beta$ . The objective function for the planning horizon  $K$  can then be formulated as follows:

$$\min_{r_k} E \left\{ \sum_{k=0}^{K-1} \alpha (v_k - v_T^k)^2 + \beta (r_k - r_T^k) | v_0 \right\} \quad (27)$$

We should note that decision  $r_k$  is upper level information that will be given to the operator of the dam. Within the constraints imposed by this decision the operator will then decide the amount of water released for hydroelectric power. For further details of operational tradeoffs in reservoir control the reader is referred to Georgakakos.<sup>17</sup> Thus there are two levels for the decisions. The first level involves the decision about the total release from the reservoir. This level of decision is provided by the LQ algorithm and is discussed in this study. The second-level decision is made to divide the total release into two parts, i.e., a release to produce power and a “nonpower” release. This step is taken by the reservoir operator who gets the first-level decision results. In order to fit the physical system into the LQ framework of Bertsekas,<sup>15</sup> Kirk,<sup>13</sup> and Athans<sup>14</sup> the following assumptions are made:

1. Let  $w_k = f_k + y_k - e_k$  be the random variable describing the joint probability of inflow, evaporation, and precipitation. Assume that  $\{w_k\}$ 's are independent, identically distributed with  $E(w_k) = \mu$ ,  $Var(w_k) = \sigma^2$ . This assumption is not generally true for hydro/climatological systems, but as Yakowitz<sup>1</sup> mentioned, the time scale and different climate zones play important roles on the independence assumption. He gives the example of summer thunderstorms in the southwest United States (sometimes referred also as "summer monsoon"), where the flows appear to have fairly short memories.
2. The physical system is capacity constrained, whereas the LQ problem formulated by equations (16)–(21) is an unconstrained problem. Therefore proceeding formally the problem is first solved without constraints. After getting the optimal policy feasibility has to be checked for the constrained problem. If the constraints are satisfied by the unconstrained LQ solution, then the solution is optimal. Otherwise the unconstrained solution has to be adjusted in order to satisfy the constraints. This can be done by "truncating" the non-fitting variables at their threshold. As mentioned by McLaughlin and Velasco,<sup>12</sup> this procedure is fairly reasonable for large reservoirs with stable storage and release levels. Therefore unconstrained optimization will be performed.

Hence, proceeding with an unconstrained optimization, the objective given in equation (27) subject to the system in equation (26) can be written into the following form to fit the LQ problem structure

$$\min_{\bar{r}_k} E \left\{ \sum_{k=0}^{K-1} \alpha_k (\bar{v}_k - \bar{v}_T^k)^2 + \beta_k \bar{r}_k^2 | \bar{v}_0 \right\} \quad (28)$$

subject to

$$\bar{v}_{k+1} = \bar{v}_k + \bar{w}_k - \bar{r}_k \quad (29)$$

where

$$\bar{v}_k = v_k - k(\bar{w} - r_T) \quad (30)$$

$$\bar{v}_T^k = v_T - k(\bar{w} - r_T) \quad (31)$$

$$\bar{r}_k = r_k - r_T \quad (32)$$

$$\bar{w}_k = w_k - \bar{w} \quad (33)$$

where  $\bar{w}$  is the mean of  $\{w_k\}$ . This is a tracking problem, which leads to a similar solution given for the general LQ model in Sections 2.1 and 2.2. Note that the transformed random variable  $\bar{w}_k$  has a zero mean. The optimum policy is obtained as follows:

$$\bar{r}_k = \mu^*(\bar{v}_k) = L_k(\bar{v}_k - \bar{v}_T^k) \quad (34)$$

Solving equations (8)–(10) with  $A_k = 1$ ,  $B_k = -1$ ,  $Q_k = \alpha_k$ , and  $R_k = \beta_k$ ,  $L_k$  and the optimal policies can be obtained. For reservoir control problems it is reasonable

to assume further that within given periods in the hydrological cycle (e.g., seasons and months) target values and objective function weights can be taken as constant, i.e.,  $v_T^k = v_T$ ,  $r_T^k = r_T$ ,  $\alpha_k = \alpha$ , and  $\beta_k = \beta$ , and stationary policies apply. In this case solving equation (12) yields

$$\frac{M^2}{M + \beta} = \alpha \Rightarrow M = \frac{\alpha + \sqrt{\alpha^2 + 4\alpha\beta}}{2} \quad (35)$$

Similarly equation (14) results in:

$$L = \frac{M}{M + \beta} \quad (36)$$

The optimum policy is obtained accordingly as follows:

$$\bar{r}_k = \mu^*(\bar{v}_k) = L(\bar{v}_k - \bar{v}_T^k) = L(v_k - v_T) \quad (37)$$

### 3.3. Computational experiment: Case study

The investigation area for the case study is Tenkiller Ferry Lake at the Illinois River basin in Oklahoma (Figure 1). The Illinois River basin is about 121 km long and has a maximum width of about 56 km. The drainage area above Tenkiller Ferry dam is about 97% of the Illinois River basin. Construction of Tenkiller Ferry Lake was initiated in June 1947. Installation of two hydropower units was completed in December 1953.

Releases of water from Tenkiller Lake are generally made through the penstock and turbines for the generation of power. During flood control operation releases are made through the spillway or outlet works conduit, and during low-flow conditions when hydropower releases are

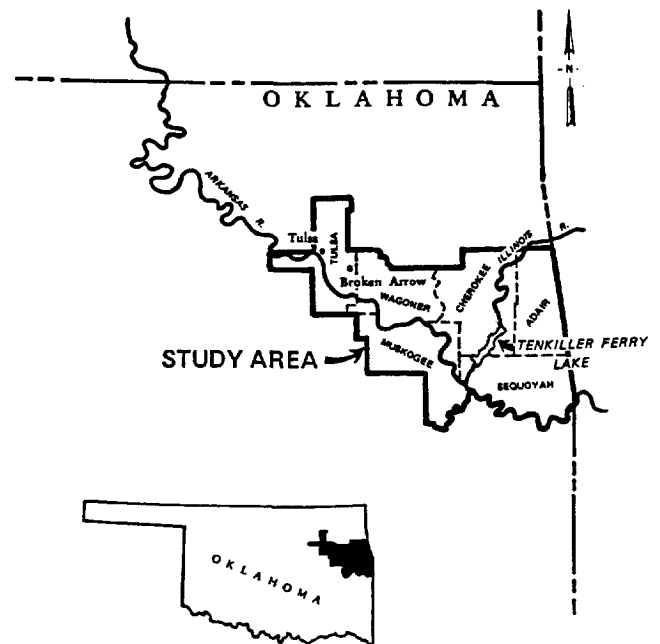


Figure 1. The study area in Oklahoma.

not needed, flows are released through the outlet works conduit. The release rate depends on power requirements, navigation water requirements, inflow rate, the amount of water in storage, riverflow downstream, and weather conditions. Other pertinent data related to the lake can be found in a study of the US Army Corps of Engineers.<sup>18</sup>

In order to study the feasibility of the LQ model 11 years (1979–1989) of daily reservoir release, storage level, inflow, precipitation, and evaporation data have been obtained from the US Army Corps of Engineers (Fort Belvoir, Virginia). The corresponding parameter values selected for the Tenkiller reservoir system are given in Table 1. These parameter values are selected as follows: For each month target reservoir releases and storage levels are computed as the monthly mean values. Similarly minimum and maximum allowed releases are determined from the available data. The minimum release turned out to be  $r_{min} = 0$  for all months. In Table 1  $\bar{w}$  denotes the average values for the joint stochastic variable (inflow, precipitation, and evaporation). The storage capacity of the Tenkiller Reservoir lake is  $v_{max} = 659 \times 10^6 \text{ m}^3$ .

If a *prescriptive* analysis is to be performed, then values for  $\alpha$  and  $\beta$  must be selected. These values are usually selected by management or by the operator. We choose instead to do a *descriptive* analysis by assuming that the historical data indeed fit an LQ model and thus by performing a model (or system) identification. For example an “indirect identification” can be performed by estimating  $\alpha$  and  $\beta$  from the available data and then by using these parameters to obtain policy values through equations (35)–(37). On the other hand the policy dictated by  $L$  can be “directly identified” from the available data by using least squares estimation (LSE) as:

$$\hat{L} = (V'V)^{-1}V'R \tag{38}$$

where  $\hat{L}$  is used to denote the estimator of  $L$ ,  $V = (v_1 - v_T, \dots, v_K - v_T)'$  and  $R = (r_1 - r_T, \dots, r_K - r_T)'$ . Furthermore algebraic manipulation of equations (35) and (36) gives:

$$\beta = \left( \frac{1 - \hat{L}}{\hat{L}^2} \right) \alpha \tag{39}$$

The LSE estimates for  $L$  in each month are given in Figure 2. It is interesting to note that the seasonality of  $\hat{L}$  in Figure 2, which shows four operating policy periods within a calendar year (1, November, December, and January; 2, February, March, and April; 3, May, June, and July; and 4, August, September, and October). Next, as a validation step using  $\hat{L}$ , simulations were performed. The comparison of both the simulated and observed reservoir release and storage series are shown in Figure 3. This figure shows that simulation and observation resemble each other fairly well. The correlation coefficients for observed and simulated release and storage series turn out to be 0.68 and 0.83, respectively. Thus the hypothesis that the operation was done according to an LQ model seems plausible. Note however that the model used for actual decision making may not be close to an original LQ model. Next for different values of  $\alpha$  corresponding  $\beta$  are estimated using equation (39). Figure 4 shows the  $L$  function for different values of  $\alpha$  and  $\beta$ . Using different  $\alpha$  and  $\beta$  scenarios the objective functions for both simulation and observation are computed. The results for 100 different  $\alpha$  ( $\alpha = 0.1, 0.2, \dots, 10$ ) and corresponding  $\beta$  combinations are shown in Figure 5. As seen from Figure 5 the optimal LQ model yields lower objective function values than observations. This shows that actually using an optimal LQ policy may yield significantly better results.

#### 4. Summary and conclusions

After presenting an alternative solution for the LQ policy tracking problem the results are applied to obtain the daily management policy of a water dam. The advantage of the LQ model is the relative simplicity of the linear optimal feedback policy and the ease of the numerical computations. The Tenkiller Dam at the Illinois River basin in Oklahoma has been analyzed as a case study. The objective is taken as a quadratic function to keep the releases and the reservoir storage at prespecified targets, which can in general be assessed from the contracted hydroelectric power and water demand for agricultural, industrial, and recreational needs.

**Table 1.** Parameters and joint stochastic variable statistics used for the LQ model

Month	$r_T$ ( $10^6 \text{ m}^3$ )	$v_T$ ( $10^6 \text{ m}^3$ )	$r_{max}$ ( $10^6 \text{ m}^3$ )	$\bar{w}$ ( $10^6 \text{ m}^3$ )
Jan.	1.07	346.92	6.32	1.57
Feb.	0.84	346.71	4.26	1.99
March	1.15	360.63	5.51	2.94
April	1.54	363.76	6.27	2.77
May	0.76	351.49	2.66	1.79
June	0.88	365.56	3.83	1.71
July	0.46	350.37	2.51	0.29
Aug.	0.37	338.46	1.59	0.31
Sep.	0.26	326.53	1.57	0.45
Oct.	0.20	326.48	2.02	0.73
Nov.	0.66	343.34	3.83	1.75
Dec.	1.07	358.95	6.23	2.70

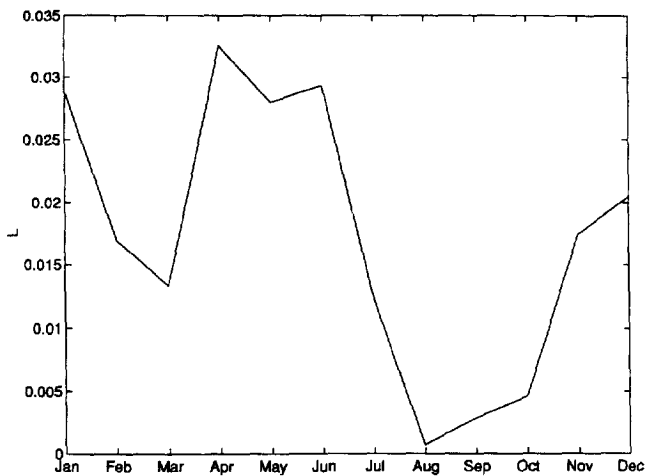


Figure 2. Least squares estimator for  $L$ .

During the simulation the minimum and maximum allowed limits for the reservoir release are violated about 5.7% of the whole time horizon, whereas the storage was between the minimum and maximum allowed limits all the time. In general, the LQ model presented herein does not guarantee the feasibility of these constraints and would result in suboptimal policies. Nevertheless, as shown in Figure 5, the LQ model gave lower objective function values than observations for all objective function weight scenarios, which shows that the proposed LQ model is

fairly robust and the use of an optimal LO policy may yield significantly better results. However we should note that we do not know the true water release and storage target values used by the management during the period of available data, which makes it difficult to compare observations and simulation.

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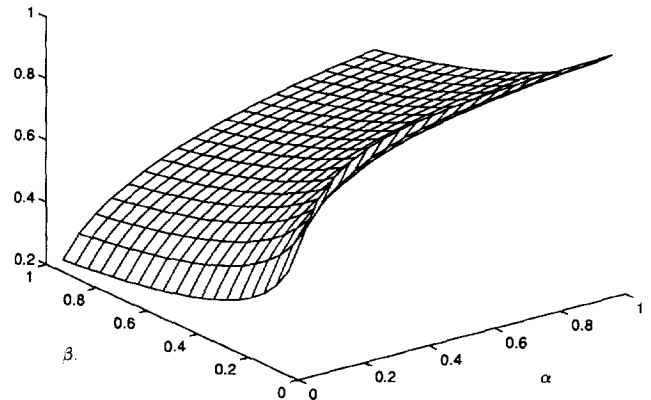


Figure 4. Plot of  $L$  with respect to  $\alpha$  and  $\beta$ .

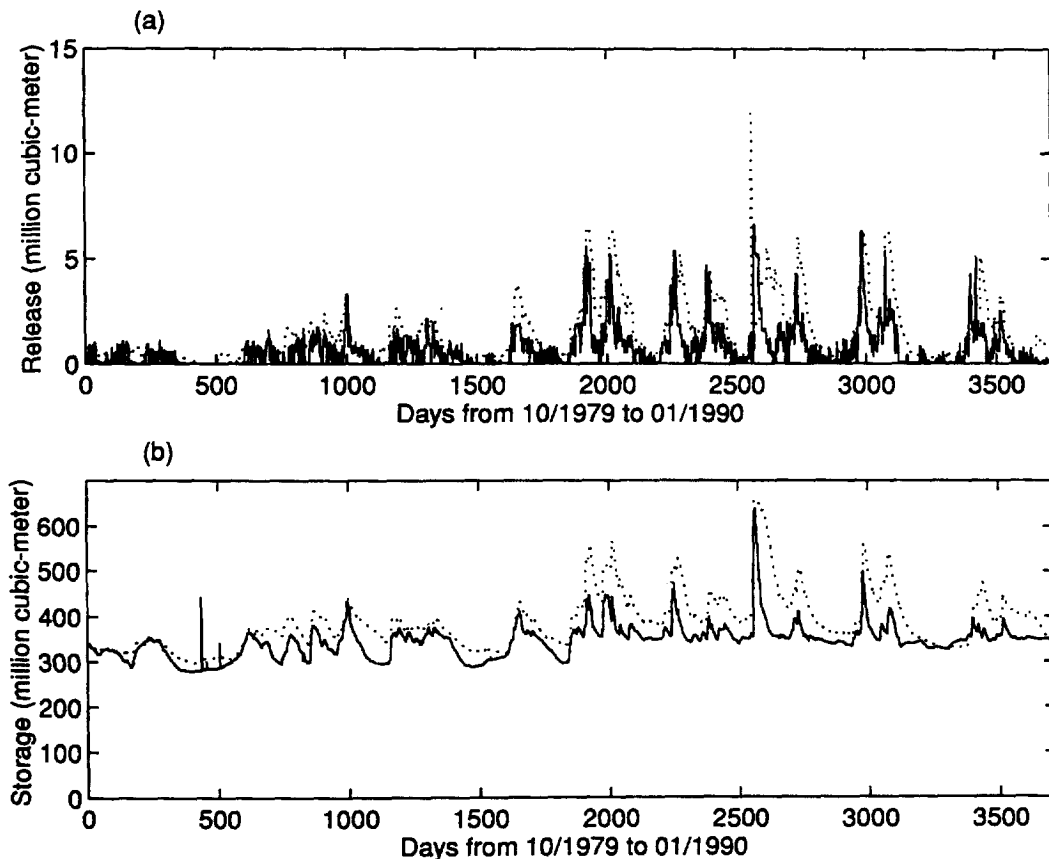


Figure 3. Comparison of an observed (solid line) and a simulated (dotted line) reservoir for (a) release and (b) storage.

