A probabilistic approach to generation maintenance scheduler with network constraints

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Abstract

Most generating unit maintenance scheduling packages consider the preventive maintenance schedule of generating units over a one or two year operational planning period in order to minimize the total operating cost while satisfying system energy requirements and maintenance constraints. In a global maintenance scheduling problem, we propose to consider network constraints and generating unit outages in generation maintenance scheduling. The inclusion of network constraints in generating unit maintenance will increase the complexity of the problem, so we decompose the global generator scheduling problem into a master problem and sub-problems using Benders decomposition. At the first stage, a master problem is solved to determine a solution for maintenance schedule decision variables. In the second stage, sub-problems are solved to minimize operating costs while satisfying network constraints and generators' forced outages. Benders cuts based on the solution of the sub-problem are introduced to the master problem for improving the existing solution. The iterative procedure continues until an optimal or near optimal solution is found.

Keywords: Generation maintenance scheduling; Network constraints; Optimization models; Decomposition techniques; Probabilistic methods

1. Introduction

Additional competition in a deregulated system and growing complexity in power generating systems, as well as a need for high service reliability and low production costs, are provoking additional interests in automatic scheduling techniques for maintenance of generators, transmission and related equipment, capable of providing least cost maintenance schedules. It should be emphasized here that the proposed model in this paper applies to both traditional utility system as well as a deregulated system. In the case of the deregulated system, we only consider a Genco’s generating units and the corresponding transmission system refers to local transmission system in a Genco which interconnects Genco’s generating units.

In the earlier works of thermal generator maintenance scheduling, most techniques were based on heuristic approaches. These approaches considered generating unit separately in selecting its optimal outage interval subject to constraints and an objective criterion for equalizing or leveling reserves throughout the planning interval [1], minimizing expected total production costs [6,10] or leveling the risk of failure to meet demand [2]. An example for a heuristic approach would be to schedule one unit at a time beginning with the largest and ending with the smallest. Most methods, mainly those based on heuristics, represent only the generation system and do not take into account network constraints effects on generation maintenance. Recently in Ref. [13] transmission constraints were represented, however, it did not recognize transmission outages.

Because of the discrete nature of maintenance scheduling, mathematical programming approaches have fallen into two broad categories: integer programming (branch and bound) [4,5] and dynamic programming [3]. More recently, Benders decomposition has been applied to decompose the problem into a master problem and a series of sub-problems [10,14].

This paper extends the Benders decomposition to include network constraints in the maintenance scheduling problem. The network is modeled as a probabilistic problem to include the effect of generation and transmission outages. Section 2 describes the formulation of maintenance scheduling with network constraints. Section 3 describes the approach for the maintenance problem using Benders decomposition. The network model used in the proposed method, the maintenance sub-problem and operation sub-problem are discussed in Section 4. The results for a simple example and the application of the proposed method to IEEE-RTS [11] are presented in Sections 5 and 6.

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2. Problem description

The maintenance scheduling problem is to determine the period for which generating units of an electric power utility should be taken off line for planned preventive maintenance over the course of a one or two year planning horizon in order to minimize the total operating cost while system energy, reliability and a number of other constraints are satisfied.

Generating units are distributed in different regions and interconnected by transmission lines. This may lead to different composite reliability levels for a given amount of maintenance capacity outage. Furthermore, generating unit maintenance schedule should consider generating unit and transmission forced and planned outages. When network constraints are included, the problem becomes considerably more complex and will be referred to as the network constrained maintenance scheduling problem. The methodology for the solution of this problem is discussed in this paper.

In the network constrained maintenance scheduling problem, the objective is to minimize the total operating cost over the operational planning period, subject to unit maintenance and operational constraints. There are two approaches which can be employed to represent generation maintenance. These approaches are fictitious cost and maintenance window. Due to difficulties in obtaining “field proven” maintenance costs, fictitious costs are often used to penalize deviations from an ideal maintenance schedule representing the preferred schedule from the power plant point of view. In maintenance window, the preferred schedule is represented by time interval (windows) and the objective is to minimize the real maintenance cost instead of fictitious cost. In this paper, we use maintenance window approach.

In order to calculate the maintenance schedule for a practically implementable schedule, numerous and complex constraints which limit the choice of scheduling times are incorporated into the solution method. The constraints in maintenance scheduling problem are categorized as coupling and decoupling constraints.

2.1. Coupling constraints

The first requirement is that units are overhauled regularly. This is necessary to keep their efficiency at a reasonable level, keep the incidence of forced outages low, and prolong the life of units and lines. This procedure is incorporated periodicity by specifying min/max times that a generating unit may run without maintenance.

The time required for overhaul is generally known, and hence the number of weeks that a machine is “down” is predetermined. It is assumed here that there is little flexibility in manpower usage that varies the time required for maintenance. Furthermore, only a limited number of machines may be serviced at once due to limited manpower.

2.2. Decoupling constraints

Network constraints in each time period are considered as decoupling constraints; the network can be modeled as either the transportation model or a linearized power flow model. We use transportation model to represent system operation limits, peak load balance equation, and generating and line capacity limits. In order to avoid over-optimistic planning, generation and transmission outages should also be taken into account (composite reliability evaluation).

Mathematically, unit maintenance scheduling can be formulated as follows:

\[
\begin{align*}
\text{Min} & \sum_t \sum_i c_{ti} y_{ti} (1-x_{ti}) + E\left\{ \sum_t \sum_i c_{ti} g_{ti} \right\} \\
\text{S.T.} & \text{maintenance constraints:} \\
x_{ti} = 1 & \text{for } t \leq e_i \text{ or } t \geq l_i + d_i \quad (i) \\
x_{ti} = 0 & \text{for } s_i \leq t \leq s_i + d_i \\
x_{ti} = 0 \text{ or } 1 & \text{for } e_i \leq t \leq l_i \\
1. \text{crew availability} & \quad 3. \text{seasonal limitations} \\
2. \text{resources availability} & \quad 4. \text{desirable schedule} \\
\text{system constraints:} & \\
S_f + g + r = d & \forall t \quad (iii) \\
g \leq \bar{g}(\phi) \cdot x & \forall t \quad (iv) \\
r \leq \bar{d} & \forall t \quad (v) \\
|f| \leq \bar{f}(\phi) & \forall t \quad (vi) \\
E\left\{ \sum_i r_{ti} \right\} \leq \varepsilon & \forall t \quad (vii)
\end{align*}
\]

where \(E\) the expected value; \(c_{ti}\) the generation maintenance cost for unit \(i\) at time \(t\); \(c_{ti}\) the generation cost of unit \(i\) at time \(t\); \(x_{ti}\) the unit maintenance status, 0 if unit is off-line for maintenance; \(s_i\) the period in which maintenance of generating unit \(i\) starts; \(e_i\) the earliest period for maintenance of generating unit \(i\) to begin; \(l_i\) the latest period for maintenance of generating unit \(i\) to begin; \(d_i\) the duration of maintenance for generating unit \(i\); \(r\) the vector of dummy generators which corresponds to energy not served at time period \(t\); \(\bar{f}\) the maximum line flow capacity in matrix term; \(f\) the active power flow in vector term; \(\bar{g}\) the maximum generation capacity in vector term; \(g\) the vector of \((g_{ti})\) power generation for each unit at time \(t\); \(d\) the vector of the demand in every bus at time \(t\); \(S\) the node-branch incidence matrix; \(\phi\) the probabilistic vector that defines the state of the system; \(\varepsilon\) the acceptable level of expected energy not served.

The unknown variable \(x_{ti}\) in (1) is restricted to integer...
values, on the contrary, \( g_e \) has continuous values. Therefore (1) corresponds to a mixed-integer programming problem.

The objective of (1) is to minimize total maintenance and production costs over the operational planning period. The production cost is a probabilistic optimization which takes into account the derated capacity of each generating unit. Constraints (i) represent the maintenance window stated in terms of the start of maintenance variables \( s_i \). The unit must not be in maintenance before its earliest period of maintenance \( (e_i) \) and its latest period of maintenance \( (e_i + d_i) \). Set of constraints (ii) consists of crew and resource availability, seasonal limitation, desirable schedule and other constraints such as environmental and fuel constraints. The seasonal limitations can be incorporated into \( e_i \) and \( d_i \) of constraint (i).

Constraints (iii)–(vi) represent peak load balance, and other operational constraints such as generation and transmission capacity limits in each state, \( \phi \), of the system. Constraint (vii) represents the reliability requirement which takes into account all states of the system. In hydro plants, operational constraints are not completely decoupled as the hydrothermal schedule usually is on chronological simulation of the generation system. For this case, one can determine additional constraints in (1) namely the energy production constraint. This constraint specifies the number of units necessary to produce a certain amount of energy or to avoid spillage in a hydro plant. This constraint can be constructed using hydrothermal simulation program [14].

Constraints (iii)–(vii) may be seen as a probabilistic problem. On the contrary, constraints (i) and (ii) represent a deterministic problem. Problem (1) has a natural structure that enables it to be decomposed into a maintenance master problem which is a deterministic integer problem and a set of operation sub-problems which are probabilistic linear programs.

### 3. Benders decomposition

Before we discuss the solution methodology, let us present the Benders decomposition by considering the following general mixed integer program:

\[
\text{Min } P \mathbf{x} + p(\mathbf{g})
\]

S.T. \( A_1 \mathbf{x} \leq \mathbf{b}_1 \)

\( A_2 \mathbf{x} + u(\mathbf{g}) \leq \mathbf{b}_2 \)

\( g \geq 0 \)

\( x_i = 0 \) or 1 for all \( i \)

where \( \mathbf{x} \) is a vector of 0–1 variables with constant cost vector \( P \) and coefficient matrices \( A_1 \) and \( A_2 \); \( g \) is a vector of continuous variables with cost functions \( p \) and \( u \); \( \mathbf{b}_1 \) and \( \mathbf{b}_2 \) are vectors of right hand side constants [7–9]. As the problem involves both discrete and continuous variables, it is unlikely that a direct approach to solve (2) would be computationally feasible. Instead we partition the problem as

\[
\text{Min } P \mathbf{x} + \min_{g \geq 0} \{ p(\mathbf{g}) | u(\mathbf{g}) \leq \mathbf{b}_2 - A_2 \mathbf{x} \}
\]

S.T. \( A_1 \mathbf{x} \leq \mathbf{b}_1 \)

\( x_i = 0 \) or 1

\( g \geq 0 \)

where \( \Omega \) is the set of \( \mathbf{x} \) for which the constraints \( u(\mathbf{g}) \leq \mathbf{b}_2 - A_2 \mathbf{x} \) can be satisfied.

For each fixed \( \mathbf{x} \) the resulting inner minimization problem is

\[
\text{Min } p(\mathbf{g})
\]

S.T. \( u(\mathbf{g}) \leq \mathbf{b}_2 - A_2 \mathbf{x} \)

\( g \geq 0 \)

The Lagrangian relaxation of (4) is given by

\[
L(\alpha) = \min_{g \geq 0} \{ p(\mathbf{g}) + \alpha(u(\mathbf{g}) - (\mathbf{b}_2 - A_2 \mathbf{x})) \}
\]

If \( \mathbf{g} \) does satisfy \( u(\mathbf{g}) \leq \mathbf{b}_2 - A_2 \mathbf{x} \), the extra term in the objective will be non-positive and thus, for all \( \alpha \geq 0 \),

\[
L(\alpha) \leq \min_{g \geq 0} \{ p(\mathbf{g}) | u(\mathbf{g}) \leq (\mathbf{b}_2 - A_2 \mathbf{x}) \}
\]

The Lagrangian dual \( L \) is then defined by \( L = \max_{\alpha \geq 0} L(\alpha) \). Under certain conditions sufficient for strong duality

\[
L = \min_{g \geq 0} \{ p(\mathbf{g}) | u(\mathbf{g}) \leq (\mathbf{b}_2 - A_2 \mathbf{x}) \}
\]

enabling us to replace the inner minimization of (3) by \( L \). This replacement is justified later for our problem. With this replacement, (3) becomes

\[
\text{Min } P \mathbf{x} + \max_{\alpha \geq 0} \{ L(\alpha) \}
\]

S.T. \( A_1 \mathbf{x} \leq \mathbf{b}_1 \)

\( x_i = 0 \) or 1

\( g \geq 0 \)

If we let

\[
\zeta = P \mathbf{x} + \max_{\alpha \geq 0} \{ L(\alpha) \} = P \mathbf{x} + \max_{\alpha \geq 0} \{ \min_{g \geq 0} \{ p(\mathbf{g}) + \alpha(u(\mathbf{g}) - (\mathbf{b}_2 - A_2 \mathbf{x})) \} \}
\]

then (8) is equivalent to

\[
\text{Min } \zeta
\]

S.T. \( A_1 \mathbf{x} \leq \mathbf{b}_1 \)

\( \zeta \geq P \mathbf{x} + \min_{g \geq 0} \{ p(\mathbf{g}) + \alpha(u(\mathbf{g}) - (\mathbf{b}_2 - A_2 \mathbf{x})) \} \)
for all $\alpha$
\[ x_i = 0 \text{ or } 1 \]
\[ x \in \Omega \]

Constraints (9) are referred to as feasibility cuts.

To complete the derivation of the master problem we must characterize the set of constraints that ensure $x \in \Omega$. This condition is satisfied if and only if
\[
\max_{\beta \geq 0} \left\{ \min_{g \geq 0} \{ p(g) + \beta(u(g) - (b_2 - A_2x)) \} \right\} \leq \infty
\]

This condition is equivalent to
\[
\min_{g \geq 0} \{ \beta(u(g) - (b_2 - A_2x)) \} \leq 0 \quad \text{for all } \beta \geq 0 \quad (10)
\]

Constraints of this form are referred to as infeasibility cuts.

Thus our master problem is
\[
\text{Min } z \quad (11)
\]
\[ S.T. \ z \geq \mathbf{P}x + \min_{g \geq 0} \{ p(g) + \alpha(u(g) - (b_2 - A_2x)) \} \]

for all $\alpha \geq 0$
\[ \min_{g \geq 0} \{ \beta(u(g) - (b_2 - A_2x)) \} \leq 0 \quad \text{for all } \beta \geq 0 \]
\[ x_i = 0 \text{ or } 1 \]
The application of this method to our problem is discussed in the following section.

4. Solution methodology

We employ Benders decomposition for the generation maintenance problem. If we let $X$ denote the vector of maintenance variables $\{x_{ik}\}$, $\Omega$ represent the set of maintenance schedules for which constraints (iii)–(vii) are satisfied in all periods $t$, we define expected operation cost $w_t$ as
\[ w_t = \sum_i E\{c_{it}g_{it}\} \]
then (1) can be written as
\[
\text{Min } \sum_t \sum_i C_{it}(1 - x_{it}) + \sum_t \min \{w_t\|(iii)-(vii)\} \quad (12)
\]
S.T.

maintenance constraints:
\[ x_{it} = 1 \quad \text{for } t \leq e_i \text{ or } t \geq l_i + d_i \]
\[ x_{it} = 0 \quad \text{for } s_i \leq t \leq s_i + d_i \quad (i) \]
\[ x_{it} = 0 \text{ or } 1 \quad \text{for } e_i \leq t \leq l_i \]

1. crew availability 3. seasonal limitations
2. resources availability 4. preschedule (ii)
\[ X \in \Omega \]

If the $r$th sub-problem was a linear program, it could be replaced by its dual as is done in the standard Benders decomposition. The Lagrangian dual of the $r$th sub-problem is given by
\[
L_r = \max_{\kappa, \pi, \gamma, \xi, \mu \geq 0} \{ L_r(\kappa, \pi, \gamma, \xi, \mu) \} \quad (13)
\]
where $L_r(\kappa, \pi, \gamma, \xi, \mu)$ is the Lagrangian function and $\kappa, \pi, \gamma, \xi, \mu$ are multipliers of constraints (iii)–(vii).

\[
L_r(\kappa, \pi, \gamma, \xi, \mu) = \min_{g \geq 0} \left\{ w_t + \sum_i \kappa_i \left( \left( \sum_k S_{ik}f_{kt} \right) + g_{it} \right) + r_{it} - d_{it} \right\} + \sum_i \pi_i (g_{it} - \bar{g}_i x_{it}) + \sum_i \gamma_i (\hat{r}_{it} - \hat{d}_i) + \sum_i \xi_i (|f_{it}| - f_k) + \mu_i \left( \left( \sum_i \hat{r}_i \right) - \varepsilon \right) \right\} \quad (14)
\]
The $r$th sub-problem is then replaced by $L_r$, and (12) is rewritten as
\[
\text{Min } \sum_t \sum_i C_{it}(1 - x_{it}) + L_r \quad (15)
\]
S.T.
maintenance constraints:
\[ x_{it} = 1 \quad \text{for } t \leq e_i \text{ or } t \geq l_i + d_i \]
\[ x_{it} = 0 \quad \text{for } s_i \leq t \leq s_i + d_i \quad (i) \]
\[ x_{it} = 0 \text{ or } 1 \quad \text{for } e_i \leq t \leq l_i \]

1. crew availability 3. seasonal limitations
2. resources availability 4. preschedule (ii)
\[ X \in \Omega \]

To ensure $X \in \Omega$, the maintenance schedule must ensure that sufficient reserve exists to provide a secure supply while minimizing the cost of operation. The $r$th sub-problem is feasible if and only if the optimal value of the following problem is less than $\varepsilon$
\[
\text{Min } E \left\{ \sum_t r_{it} \right\} \quad (16)
\]
S.T.

\[ Sf + g + r = d \]
\[ g \leq \tilde{g}(\varphi)X \]
\[ r \leq d \]
\[ |f| \leq \tilde{f}(\varphi) \]
Its Lagrangian dual is:
\[
\max_{\nu, \lambda, \tau, \eta} U(\nu, \lambda, \tau, \eta)
\]
where \(U(\nu, \lambda, \tau, \eta)\) is the following Lagrangian function and \(\nu, \lambda, \tau\) and \(\eta\) are multipliers of constraints (iii)–(vii).

\[
U(\nu, \lambda, \tau, \eta) = \min_{g \geq 0} \left\{ \sum_i r_{it} + \sum_i \nu_i \left( \sum_k S_{ik} f_{kt} \right) + g_a + r_a - d_{it} + \sum_i \lambda_i (g_{it} - \bar{g}_i \cdot x_{it}) + \sum_i \tau_i (r_{it} - d_{it}) + \sum_k \eta_{ik} (f_{kt} - \bar{f}_k) \right\}
\]

We then arrive at the generalized Benders master problem:

Min \(z\)

S.T.

\[
z \geq \sum_i \sum_k C_{it} (1 - x_{it}) + \sum_i L_i (\kappa, \pi, \gamma, \zeta, \mu)
\]

for all \(\kappa, \pi, \gamma, \zeta, \mu \geq 0 \)

\[
\sum_i U_i(\nu, \lambda, \tau, \eta) \leq \varepsilon \quad \text{for all} \quad \nu, \lambda, \tau, \eta \geq 0
\]

maintenance constraints:

\[
x_{it} = 1 \quad \text{for} \quad t \leq e_i \text{ or } t \geq l_i + d_i
\]

\[
x_{it} = 0 \quad \text{for} \quad s_i \leq t \leq s_i + d_i \quad (i)
\]

\[
x_{it} = 0 \text{ or } 1 \quad \text{for} \quad e_i \leq t \leq l_i \quad (ii)
\]

1. crew availability 3. seasonal limitations
2. resources availability 4. desirable schedule

The problem is decomposed into a master problem and operation sub-problems. The master problem, which is an integer programming problem, is solved to generate a trial solution for maintenance schedule decision variables. This master problem is a relaxation of the original problem in that it contains only a subset of constraints. Its optimal value is a lower bound on the optimal value of original problem. Once \(x_{it}\) variable is fixed, the resulting operation sub-problem can be treated as a set of independent sub-problems, one for each time period \(t\), as there is no constraint across time periods. The set of operation sub-problems are then solved using the fixed maintenance schedule obtained from the solution of the master problem. At each iteration the solution of sub-problems generates dual multipliers, which measure the change in either production cost or reliability resulting from marginal changes in the maintenance schedule. These dual multipliers are used to form one or more constraints (known as cuts) which are
added to the master problem for the next iteration. The process continues until a feasible solution is found whose cost is sufficiently close to lower bound, as shown in Fig. 1.

The initial maintenance master problem is formulated as follows:

$$\text{Min } z$$

S.T.

$$z \geq \sum_{t} \sum_{i} \left( C_{it} g_{i} \right) (1 - x_{it})$$

maintenance constraint:

$$x_{it} = 1 \quad \text{for } t \leq e_{i} \text{ or } t \geq l_{i} + d_{i}$$

$$x_{it} = 0 \quad \text{for } s_{i} \leq t \leq s_{i} + d_{i}$$

$$x_{it} = 0 \text{ or } 1 \quad \text{for } e_{i} \leq t \leq l_{i}$$

1. crew availability 3. seasonal limitations

2. resources availability 4. desirable schedule

4.1. Operation sub-problems

If the sub-problems are feasible, then the fuel cost for period \( t \), \( w_{i} \), depends on the utilization of available units to satisfy load constraints in each time period subject to maintaining reliability above a certain level. The reliability level, \( \varepsilon(\varphi) \), is the solution of feasibility check in its associated state space. Thus, the generation cost in period \( t \) can be expressed as

$$w_{i} = \text{Min } E \left\{ \sum_{t} C_{it} g_{i} x_{it} \right\}$$

(19)

S.T. \( Sf + g + r = d(\varphi) \)

$$g \leq \tilde{g}(\varphi) \cdot x^{d} \quad \text{(dual variable is } \pi)$$

$$r \leq d(\varphi)$$

$$|f| \leq \tilde{f}(\varphi)$$

$$\sum_{t} r_{it} \leq \varepsilon(\varphi)$$

The calculation of (16) and (19) involve the reliability evaluation of a composite system. The procedure is as follows:

1. Select a system state \( \varphi \), i.e. define load levels, equipment availability, operating conditions, etc.
2. Calculate (16) or (19) for the selected state, i.e. verify whether that specific configuration of generators and transmission lines is able to supply that specific load without violating system limits.
3. Update the estimated of production cost or load shedding expectation.
4. Return to step 1 for the next system state \( \varphi \) if the accuracy is unacceptable.

In order to allow fast and efficient calculations, we use simultaneous decomposition simulation for all generator and transmission states and intervals [12]. The model generates a new linear constraint for the master problem (Benders cuts) based on linear sensitivities of loss of load to the capacities of generation units and transmission lines in the case of (16). In the case of (19), cuts are generated based on linear sensitivities of fuel cost to the capacities of generation units and transmission lines. Algorithm to find multiplier \( p \) is given in Fig. 2. To find multiplier \( \lambda \), (16) is applied instead of (19).

The solution of the sub-problem is not complicated, as knowing which generator and transmissions are available during period \( t \) allows us to minimize the expected operation cost. The feasible cut is of the form

$$z \geq \sum_{t} \left( w_{it}^{n} + \sum_{i} \left( C_{it} g_{i} (1 - x_{it}^{n}) + \pi_{it}^{n} \tilde{g}_{i} (x_{it}^{n} - x_{it}) \right) \right)$$

(20)

where \( w_{it}^{n} \) is the expected fuel cost for period \( t \) associated with the \( n \)th trial solution. The multiplier \( \pi_{it}^{n} \) may be interpreted as expected marginal costs associated with 1 MW decrease of generator \( i \) capacity, given the \( n \)th trial maintenance schedule. The cost cuts (20) will tend to increase lower bounds obtained from successive maintenance sub-problem solutions.
The sub-problem may not have any solutions due to the fact that the unserved energy cannot be kept above a desired level. If a sub-problem is infeasible, then infeasibility cut is generated. For each infeasible sub-problem resulting from the \( n \)th trial solution of the master problem, the infeasible cut is of the form

\[
E \left\{ \sum_i r_{it}^n \right\} + \sum_i \lambda_i^n \delta_i^n (x_i^n - x_i) \leq \varepsilon
\]  

(21)

The multiplier \( \lambda_i^n \) may be interpreted as the marginal decrease in energy not supplied with a 1 MW increase of generator \( i \), given the \( n \)th iteration of maintenance schedule. The infeasibility cuts (19) will eliminate maintenance values, \( x_i \), which is not possible to be scheduled.

### 4.2. Maintenance master problem

The maintenance master problem is the minimization of maintenance cost subject to maintenance constraints as well as feasibility and infeasibility cuts from the operation sub-problems. If all of operation sub-problems are feasible then their solution yield a set of dual multipliers from which a feasibility cut is constructed. If one or more of operation sub-problems are infeasible then for each infeasible sub-problem an infeasibility cut is generated

\[
\text{Min } z \leq \sum_i \sum \left\{ C_i \delta_i^n (1 - x_i) \right\}
\]  

(22)

The important feature of the Benders decomposition is the availability of upper and lower bounds to the optimal solution at each iteration. These bounds can be used as an effective convergence criterion. The critical point in the decomposition is the modification of objective function based on the solution of operation sub-problem. Associated with the solution of the operation sub-problem is a set of dual multipliers which measure changes in system operating costs caused by marginal changes in the trial maintenance. These multipliers are used to form a linear constraint, written in terms of maintenance variable \( x \). This constraint, known as Benders cut, is returned to the maintenance problem which is modified and solved again to determine a new trial maintenance plan.

### 5. Example

We use a three-bus system as an example. For convenience, in this sample study, all lines are assumed to be perfectly reliable. The forced outage rate (F.O.R.) of each generator is given in Table 1. The system reliability requirement \( (\varepsilon) \) is 0.5 p.u. The generator, line data in per unit are given in Tables 1 and 2. Load data is depicted in Fig. 3. The problem is defined as: We are to perform maintenance on at

<table>
<thead>
<tr>
<th>Line</th>
<th>( \Omega/\text{line} )</th>
<th># of lines</th>
<th>Capacity/line (p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–2</td>
<td>0.2</td>
<td>2</td>
<td>0.25</td>
</tr>
<tr>
<td>2–3</td>
<td>0.25</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>1–3</td>
<td>0.4</td>
<td>2</td>
<td>0.25</td>
</tr>
</tbody>
</table>
least one generator. We assume the study period only has one time interval. Loads are assumed constant during the study period.

First, we solve the relaxed maintenance master problem.

**Maintenance master problem iteration 1:**

Min \( z \)

S.T. \( 300 \times (1 - x_1) + 200 \times (1 - x_2) + 100 \times (1 - x_3) \leq z \)

\( x_1 + x_2 + x_3 \leq 2 \)

\( x_1 \leq 1 \quad x_2 \leq 1 \quad x_3 \leq 1 \)

The solution is:

\( x_1 = 1 \quad x_2 = 1 \quad x_3 = 0 \) and \( z = 100 \).

**Operation sub-problem iteration 1:**

We check the feasibility of operation sub-problem given the first trial of maintenance schedule. The first state space formulation of the feasibility check is as follows:

Min \( r_1 + r_2 + r_3 \)

S.T. \(-f_{12} - f_{13} + g_1 + r_1 = 1 \) Load balance at bus 1

\(-f_{23} + f_{12} + g_2 + r_2 = 3 \) Load balance at bus 2

\( f_{13} + f_{23} + g_3 + r_3 = 1 \) Load balance at bus 3

\( 0.5 \leq g_1 \leq 2.5 \) Generator 1 limit

\( 0.6 \leq g_2 \leq 2.5 \) Generator 2 limit

\( 0 \leq g_3 \leq 0 \) Generator 3 limit

\(-2 \times 0.25 \leq f_{12} \leq 2 \times 0.25 \) Line 1–2 flow limit

\(-2 \times 0.25 \leq f_{13} \leq 2 \times 0.25 \) Line 1–3 flow limit

\(-2 \times 0.5 \leq f_{23} \leq 2 \times 0.5 \) Line 2–3 flow limit

The primal solution for all state spaces of the feasibility check can be seen in Table 3. The dual price of the operation sub-problem is:

\( \lambda_{x1} = 0 \quad \lambda_{x2} = 0.9 \quad \lambda_{x3} = 0.98 \)

The above LP solution is infeasible, as \( E\{r_1 + r_2 + r_3\} \geq 0.5 \). The cost is set arbitrarily to \( w = 1000 \) because the solution is infeasible. The infeasible cut is as follows:

\( 0.85 + 0.9 \times 2.5 \times (1 + x_3) + 0.98 \times 3 \times (0 - x_3) \leq 0.5 \).

**Maintenance master problem iteration 2:**

Min \( z \)

S.T. \( 300 \times (1 - x_1) + 200 \times (1 - x_2) + 100 \times (1 - x_3) \leq z \)

\( 0.85 + 0.9 \times 2.5 \times (1 - x_2) + 0.98 \times 3 \times (0 - x_3) \leq 0.5 \)

\( x_1 + x_2 + x_3 \leq 2 \)

\( N_{12} \leq 2 \quad N_{23} \leq 2 \quad N_{13} \leq 2 \)

\( x_1 \leq 1 \quad x_2 \leq 1 \quad x_3 \leq 1 \)

The solution is:

\( x_1 = 1 \quad x_2 = 0 \quad x_3 = 1 \) and \( z = 200 \).

**Operation sub-problem iteration 2:**

We check the feasibility of operating sub-problem given the first trial of maintenance schedule. The first state space formulation of the feasibility check is as follows:

Min \( r_1 + r_2 + r_3 \)

S.T. \(-f_{12} - f_{13} + g_1 + r_1 = 1 \) Load balance at bus 1

\(-f_{23} + f_{12} + g_2 + r_2 = 3 \) Load balance at bus 2

\( f_{13} + f_{23} + g_3 + r_3 = 1 \) Load balance at bus 3

\( 0.5 \leq g_1 \leq 2.5 \) Generator 1 limit

\( 0 \leq g_2 \leq 3 \) Generator 2 limit

\( 0.6 \leq g_3 \leq 3 \) Generator 3 limit

\(-2 \times 0.25 \leq f_{12} \leq 2 \times 0.25 \) Line 1–2 flow limit

\(-2 \times 0.25 \leq f_{13} \leq 2 \times 0.25 \) Line 1–3 flow limit

\(-2 \times 0.5 \leq f_{23} \leq 2 \times 0.5 \) Line 2–3 flow limit

The primal solution for all state spaces of feasibility check can be seen in Table 4. The dual price of the operation sub-problem is: \( \lambda_{x1} = 0 \quad \lambda_{x2} = 0.9 \quad \lambda_{x3} = 0 \). The above LP solution is infeasible, as \( E\{r_1 + r_2 + r_3\} \geq 0.5 \). The cost is set arbitrarily to \( w = 1000 \) because the solution is infeasible.
Generating unit operating cost data

Table 6
Generating units considered

<table>
<thead>
<tr>
<th>Unit</th>
<th>Capacity (MW)</th>
<th>Bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 × 76</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2 × 76</td>
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</tr>
<tr>
<td>3</td>
<td>1 × 100</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>2 × 100</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>2 × 20</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 7
Generating operating cost data

<table>
<thead>
<tr>
<th>Size</th>
<th>Fuel</th>
<th>Heat rate</th>
<th>Maintenance cost</th>
<th>Maintenance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Fixed (10^3 US$/Yr.)</td>
<td>Variable (US$/kW/Yr.)</td>
</tr>
<tr>
<td>MW</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Oil #2</td>
<td>14 500</td>
<td>6.0</td>
<td>5.0</td>
</tr>
<tr>
<td>76</td>
<td>Coal</td>
<td>12 000</td>
<td>760.0</td>
<td>0.9</td>
</tr>
<tr>
<td>100</td>
<td>Oil #6</td>
<td>10 000</td>
<td>850.0</td>
<td>0.8</td>
</tr>
</tbody>
</table>
The solution is: 

\[ \begin{align*} 
\min \quad & z = 300 (1 - x_1) + 200 (1 - x_2) + 100 (1 - x_3) + 547 \times 1 - 9.5 \times 2.5 (0 - x_1) - 9 \times 2.5 (1 - x_2) \\
\text{subject to} \quad & x_1 + x_2 + x_3 \leq 2 \\
\text{and} \quad & N_{12} \leq 2, \quad N_{23} \leq 2, \quad N_{13} \leq 2 \\
\text{and} \quad & x_1 \leq 1, \quad x_2 \leq 1, \quad x_3 \leq 1 \\
\end{align*} \]

We stop here as \( z = w \) which means the cost is equal to the lower bound.

### 6. Case study

We apply the proposed method to the IEEE-RTS [11]. This system is made of 32 generating units, 20-demand sides, 23 buses and 38 transmission lines. A three-month study period of summer weeks, week 18–29, is considered. Some generation facilities in a particular area need maintenance within the study period. The coverage of this area includes buses 1–10.

Table 6 gives the generating units which are to be maintained. Table 7 gives the operating characteristics of generating units in this area. The average fuel price is US$3.00/MBtu for Oil #2, US$1.2/MBtu for coal and US$2.3/MBtu for Oil #6. Detailed system data for transmission lines, generators and loads can be seen in Appendix A.

The minimization of production cost and maintenance cost are used as the objective function. Energy not served in each week is limited to the maximum of 1% of the total energy. The results of the following test cases are included to show the effect of network constraint on maintenance schedule and unit loading points.

- Case 0: No network constraints.
- Case 1: Network constraints are considered.

In Case 0, no network constraints, the problem is the classical unit maintenance schedule. Case 0 is converged in two iterations. In the first iteration, sub-problems periods are infeasible in all time. In iteration two, sub-problems are feasible and the final cost is given in Table 8. The maximum transmission flow over the three-month study period is shown in Table 9. The corresponding generator loading points of the weekly load are shown in Table 10. In this table, the 12-week horizon is between weeks 18–29; weeks up and down refer to periods in which units are used for supplying the load. For Case 0, each line flow is within limits for most of time period except for lines 1–3 and 11–14. Here, unit 5 is not used in all time periods.

Case 1 studies the effect of transmission limits on maintenance schedule. In Case 1, transmission limits are imposed on the optimization problem. The imposed transmission limits increase the cost of operation. Table 5 shows the change in operating cost over the study period, indicating a shift from units that use inexpensive fuel to those with more expensive fuels and inefficient units.

In practice, units are loaded in decreasing order of the operating cost efficiency. With transmission limitations, available units in one time period may become less attractive when compared to those in some other time periods.

### Table 6

<table>
<thead>
<tr>
<th>Case</th>
<th>Unit</th>
<th>Weeks on maintenance</th>
<th>Weeks up</th>
<th>Weeks down</th>
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<tr>
<td>0</td>
<td>1</td>
<td>24, 25</td>
<td>18–23, 26–29</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>27, 28</td>
<td>18–26, 29</td>
<td>–</td>
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<tr>
<td></td>
<td>3</td>
<td>20, 21</td>
<td>18, 19, 22–29</td>
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<tr>
<td></td>
<td>4</td>
<td>18, 19</td>
<td>20, 21, 24, 25, 27, 28</td>
<td>22, 23, 26, 29</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>22, 23</td>
<td>–</td>
<td>18–21, 24–29</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>18, 19</td>
<td>20–29</td>
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<td></td>
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<td>26, 27</td>
<td>18–25, 28–29</td>
<td>–</td>
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<tr>
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<td>23, 24</td>
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<td></td>
<td>4</td>
<td>19, 20</td>
<td>18, 21, 23, 24, 28, 29</td>
<td>22, 25–27</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>27, 28</td>
<td>23, 24</td>
<td>18–22, 25, 26, 29</td>
</tr>
</tbody>
</table>
5 has to be brought on-line for weeks 23 and 24 to supply generation deficit. The test system was applied over a year-long horizon of 52 week period. Table 11 shows the maintenance schedule for generating units.

As the units are not forced to be maintained within 12 weeks, the cost can be reduced as given in Table 12 and a better distribution of the risk can be achieved. The generating units maintenance schedule is shifted to the lower peak load: The maintenance of unit 1 is shifted from weeks 18–19 to weeks 37–38. Also units 2–5 are shifted from weeks 26–27 to weeks 38–39, weeks 23–24 to 10–11, weeks 19–20 to 16–17, weeks 27–28 to 36–37, respectively.

7. Conclusions

This paper presents a decomposition approach based on the duality theory for generation maintenance scheduling with network constraints. The test results demonstrate that the limits on transmission line capacity affect the loading points of units and increase the generation by expensive and inefficient units, resulting in an increase in the overall cost of operation.

The extension of generation maintenance scheduling to include network constraints is applicable to the problem of maintenance with probabilistic data. Using the proposed decomposition method, additional complex constraints can be imposed on the maintenance scheduling problem.

Appendix A

A.1. System data

The transmission network consists of 24 bus locations connected by 38 lines. Impedance and rating data for lines is given in Tables A1 and A2. The location of generating units are shown in Table A3. It can be seen that 10 of 24
buses are generating stations. From these generating stations we have decided to do maintenance for only 3 generating stations at buses 1, 2 and 7 (see Table 6). The generating unit operating cost data can seen in Table A4. Table A5 gives data on weekly peak loads in percent of the annual peak load. The annual peak load for the test system is 2850 MW. The data in Table A5 shows a typical pattern, with two seasonal peaks.

Table A1 (continued)

<table>
<thead>
<tr>
<th>From bus</th>
<th>To bus</th>
<th>Impedance (p.u./100 MVA base)</th>
<th>No. of lines</th>
<th>Rating (MVA)</th>
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</thead>
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<tr>
<td></td>
<td></td>
<td>$R$</td>
<td>$X$</td>
<td>$B$</td>
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<tr>
<td>16</td>
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<td>17</td>
<td>22</td>
<td>0.0135</td>
<td>0.1053</td>
<td>0.2212</td>
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<tr>
<td>18</td>
<td>21</td>
<td>0.0033</td>
<td>0.0259</td>
<td>0.0545</td>
</tr>
<tr>
<td>19</td>
<td>20</td>
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<td>0.0396</td>
<td>0.0833</td>
</tr>
<tr>
<td>20</td>
<td>23</td>
<td>0.0028</td>
<td>0.0216</td>
<td>0.0455</td>
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<tr>
<td>21</td>
<td>22</td>
<td>0.0087</td>
<td>0.0678</td>
<td>0.1424</td>
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</table>

Table A2
Transmission line length and forced outage*

<table>
<thead>
<tr>
<th>From bus</th>
<th>To bus</th>
<th>Length (miles)</th>
<th>Outage rate (1/year)</th>
<th>Outage duration (hours)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>0.24</td>
<td>16</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>55</td>
<td>0.51</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>22</td>
<td>0.33</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>33</td>
<td>0.39</td>
<td>10</td>
</tr>
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<td>2</td>
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<td>0.48</td>
<td>10</td>
</tr>
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</tr>
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<td>3</td>
<td>24</td>
<td>0</td>
<td>0.02</td>
<td>768</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>27</td>
<td>0.36</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>23</td>
<td>0.34</td>
<td>10</td>
</tr>
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<td>6</td>
<td>10</td>
<td>16</td>
<td>0.33</td>
<td>10</td>
</tr>
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<td>7</td>
<td>8</td>
<td>16</td>
<td>0.30</td>
<td>768</td>
</tr>
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<td>9</td>
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<td>768</td>
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<td>0.44</td>
<td>768</td>
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<td>11</td>
<td>0</td>
<td>0.02</td>
<td>768</td>
</tr>
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<td>9</td>
<td>12</td>
<td>0</td>
<td>0.02</td>
<td>11</td>
</tr>
<tr>
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<td>11</td>
<td>0</td>
<td>0.02</td>
<td>11</td>
</tr>
<tr>
<td>10</td>
<td>12</td>
<td>0</td>
<td>0.02</td>
<td>11</td>
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<td>11</td>
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<td>11</td>
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</tr>
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<td>13</td>
<td>23</td>
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<td>0.49</td>
<td>11</td>
</tr>
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<td>0.38</td>
<td>11</td>
</tr>
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<td>16</td>
<td>12</td>
<td>0.33</td>
<td>11</td>
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<td>11</td>
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<td>24</td>
<td>36</td>
<td>0.41</td>
<td>11</td>
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<td>18</td>
<td>0.35</td>
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<td>0.34</td>
<td>11</td>
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<td>18</td>
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<td>18</td>
<td>0.35</td>
<td>11</td>
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<td>19</td>
<td>20</td>
<td>27</td>
<td>0.38</td>
<td>11</td>
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<td>20</td>
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</tr>
<tr>
<td>21</td>
<td>22</td>
<td>47</td>
<td>0.45</td>
<td>11</td>
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</tbody>
</table>

* Forced outage rates are calculated as

\[
\text{F.O.R.} = \frac{\text{Outage Duration}}{\text{Outage Rate} + \text{Outage Duration}}
\]

Table A3
Generating units locations

<table>
<thead>
<tr>
<th>Unit</th>
<th>Capacity (MW)</th>
<th>Bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 x 76</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2 x 76</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1 x 100</td>
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<tr>
<td>4</td>
<td>2 x 100</td>
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<tr>
<td>5</td>
<td>2 x 20</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>3 x 197</td>
<td>13</td>
</tr>
<tr>
<td>7</td>
<td>5 x 12</td>
<td>15</td>
</tr>
<tr>
<td>8</td>
<td>1 x 155</td>
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<tr>
<td>9</td>
<td>1 x 155</td>
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<td>6 x 50</td>
<td>22</td>
</tr>
<tr>
<td>13</td>
<td>2 x 155</td>
<td>23</td>
</tr>
<tr>
<td>14</td>
<td>1 x 350</td>
<td>23</td>
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</tbody>
</table>

Table A4
Generating units operating cost data

<table>
<thead>
<tr>
<th>Size MW</th>
<th>Fuel</th>
<th>Cost (US$/MBtu)</th>
<th>Heat rate (Btu/kWh)</th>
<th>F.O.R.</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>Oil #6</td>
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<td>12 000</td>
<td>0.02</td>
</tr>
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<td>20</td>
<td>Oil #2</td>
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<td>14 500</td>
<td>0.10</td>
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<td>100</td>
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<td>10 000</td>
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<td>9500</td>
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<tr>
<td>400</td>
<td>Nuclear</td>
<td>0.60</td>
<td>10 000</td>
<td>0.12</td>
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</table>
Table A5
Weekly peak load in percent of annual peak

<table>
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<th>Peak load</th>
<th>Week</th>
<th>Peak load</th>
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<td>90.0</td>
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<td>81.6</td>
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<td>87.8</td>
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<td>80.0</td>
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<td>72.0</td>
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