A Note on the Ross–Taylor Theorem

Emmanuel Fernández-Gaucherand
Systems and Industrial Engineering Department
The University of Arizona
Tucson, Arizona 85721

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ABSTRACT

In this note the conditions used in proving a result due to E. M. Ross and H. M. Taylor are examined. These results pertain to the existence of bounded solutions to the average cost optimality equation for controlled Markov processes with an average cost criterion. In particular, we show how the use of commonly found convexity (concavity) properties of value functions can be employed to verify a seemingly rather restrictive equicontinuity condition. In addition, we remark that several results in the literature can be viewed as special cases of the results by Ross and Taylor, contrary to claims otherwise.

1. INTRODUCTION

Consider a controlled Markov process (CMP) described by the quadruplet \((X, U, Q, c)\), where \(X\) is the state space, a (Borel) subset of a complete and separable metric space, \(U\) is a finite set of actions (or decisions), \(Q\) is a stochastic kernel describing the distribution of the next state \(X_{t+1}\), given the current state-action pair \((X_t, U_t)\), \(c : X \times U \to \mathbb{R}\) is the bounded one-stage cost function. A (stationary) policy is a rule \(\pi : X \to U\) for making decisions, such that \(U_t = \pi(X_t)\). We refer to [1-4] for more details on the description of the model.

Given a policy \(\pi\) and an initial state \(x_0\), two criteria commonly used to measure the performance of the system are the discounted cost (DC),

\[
J^\pi_{D}(x_0) := \lim_{N \to \infty} \mathbb{E}^\pi_{x_0}\left\{ \sum_{t=0}^{N} \beta^t c(X_t, U_t) \right\},
\]

\(1\)

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2. THE TÓSSON/TÖRÖK THEOREM

The Tósson/Török theorem is a powerful result in the theory of complex analysis. It states that if a function is holomorphic in a certain domain, then it has a certain property that allows it to be extended to a larger domain. This theorem is particularly useful in the study of complex functions and their applications. In order to prove the theorem, we need to establish several key conditions. We will now examine these conditions in detail. The main theorem is as follows:

\[ \frac{1}{\pi} \int_{|z| = R} f(z) \, dz = \frac{1}{2} (f(0) + f(2R)) \]

We begin by considering the conditions of the theorem. The theorem is divided into several parts, each of which must be satisfied for the theorem to hold. The key conditions are as follows:

- Condition 1:
  - \( f(0) \)
  - \( f(2R) \)

- Condition 2:
  - \( \frac{1}{\pi} \int_{|z| = R} f(z) \, dz \)

- Condition 3:
  - \( \frac{1}{2} \)

By satisfying these conditions, we can conclude that the theorem holds. The proof of the theorem is based on the Cauchy integral formula and the properties of holomorphic functions. The Cauchy integral formula states that if a function is holomorphic in a certain domain, then it has a certain property that allows it to be extended to a larger domain. This theorem is particularly useful in the study of complex functions and their applications.